Flux, helicity and buoyancy in protostars

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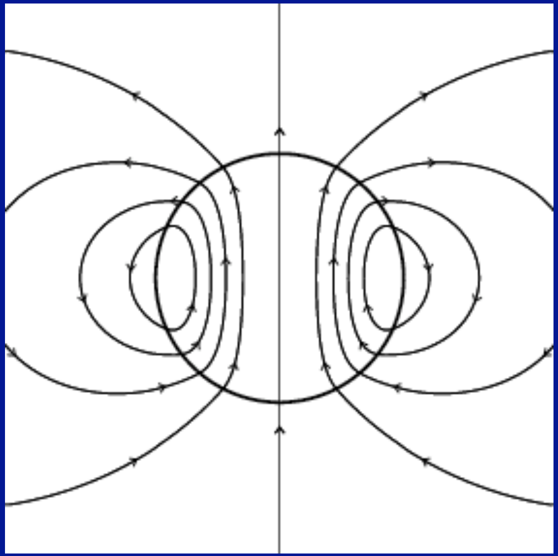
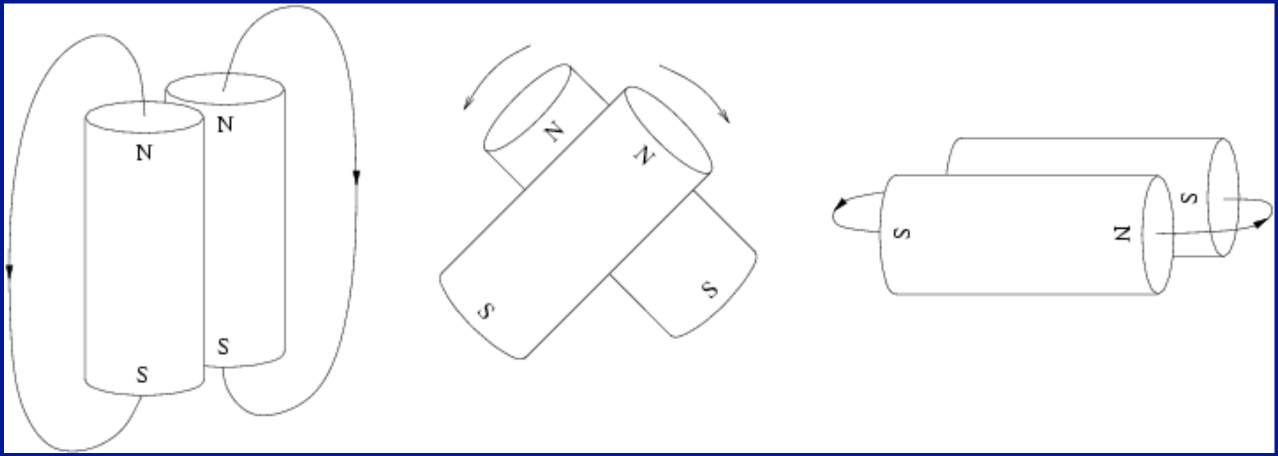
Flux problem

- Magnetic field required to form stars, and clouds contain large flux, but we observe stars with only very weak fields
- How could flux be reduced?
  - Ohmic & ambipolar diffusion (at lower densities)
  - Reconnection inside star
  - Reconnection through surface

Reconnection as means to reduce flux

- Imagine star contains a large poloidal flux
- Instability --> reconnection
  --> magnetic energy destroyed



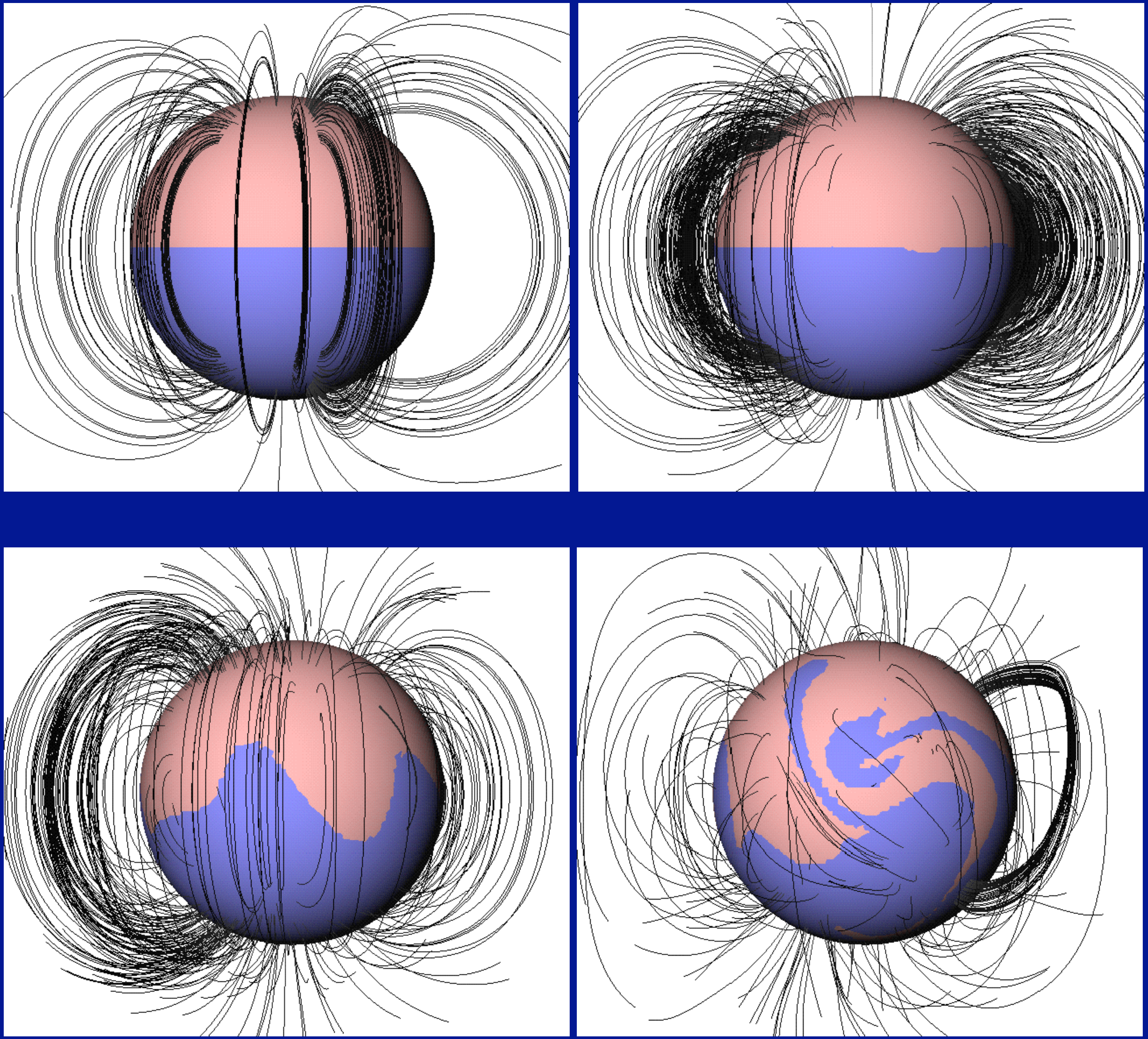


Poloidal field:
star can be thought of as fluid of aligned bar magnets

(Markey & Tayler 1973,74; Wright 1973; Flowers & Ruderman 1977)

- Reconnection ends when equilibrium reached
- How to predict equilibrium field strength in star?

Simulation of decay of poloidal field



Frames at t = 0, 4.4, 5.0 and 5.7 Alfvén crossing times

Red & blue show regions of +ve and -ve B_r

Formation of equilibria:
helicity conservation and strength of equilibrium field

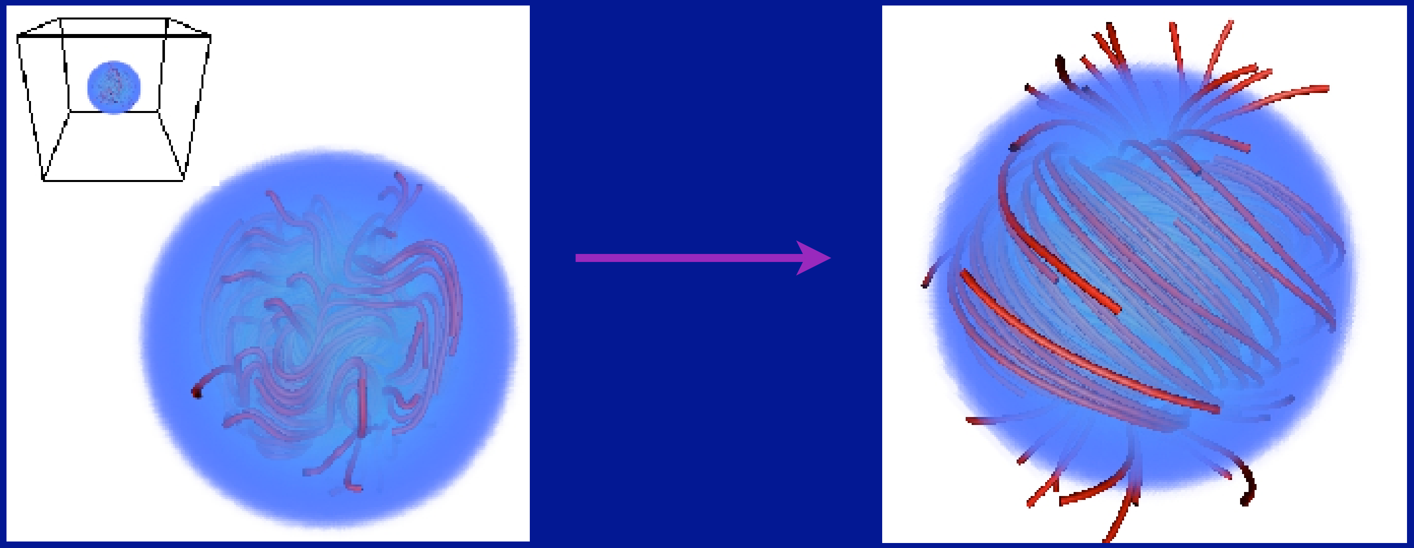
- Magnetic helicity definition: $$H \equiv \int \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V \quad \text{where} \quad \mathbf{B} = \nabla \times \mathbf{A}$$
- *H* is perfectly conserved in a fluid of infinite conductivity (Woltjer 1958) (provided that boundary is magnetic surface)
- *H* has units length x energy, so is roughly conserved while energy is dissipated on small scales
- A given equilibrium type *n* has an associated dimensionless length scale λ_n

$$\frac{H_{\mathrm{eq}}}{E_{\mathrm{eq}}} \approx \lambda_n R \qquad \text{and} \qquad H_{\mathrm{eq}} \approx H_{\mathrm{init}} \qquad \text{therefore} \qquad E_{\mathrm{eq}} \approx \frac{H_{\mathrm{init}}}{\lambda_n R}$$

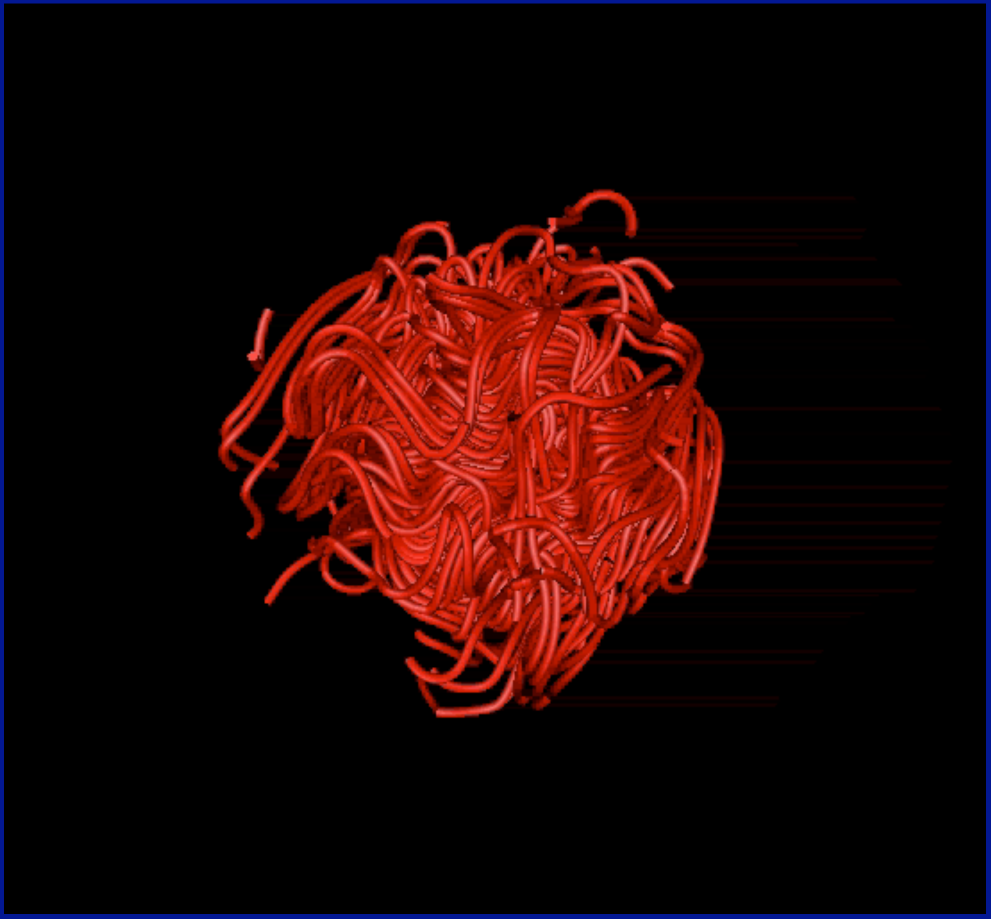
- Energy of final equilibrium (and strength of ‘fossil field’) can be predicted if we know the initial *helicity*. *Energy* (or poloidal flux) of initial field is not relevant.
- Helicity can be thought of as $H \sim \Phi_{\mathrm{pol}} \Phi_{\mathrm{tor}}$
-

Finding equilibria: numerical methods

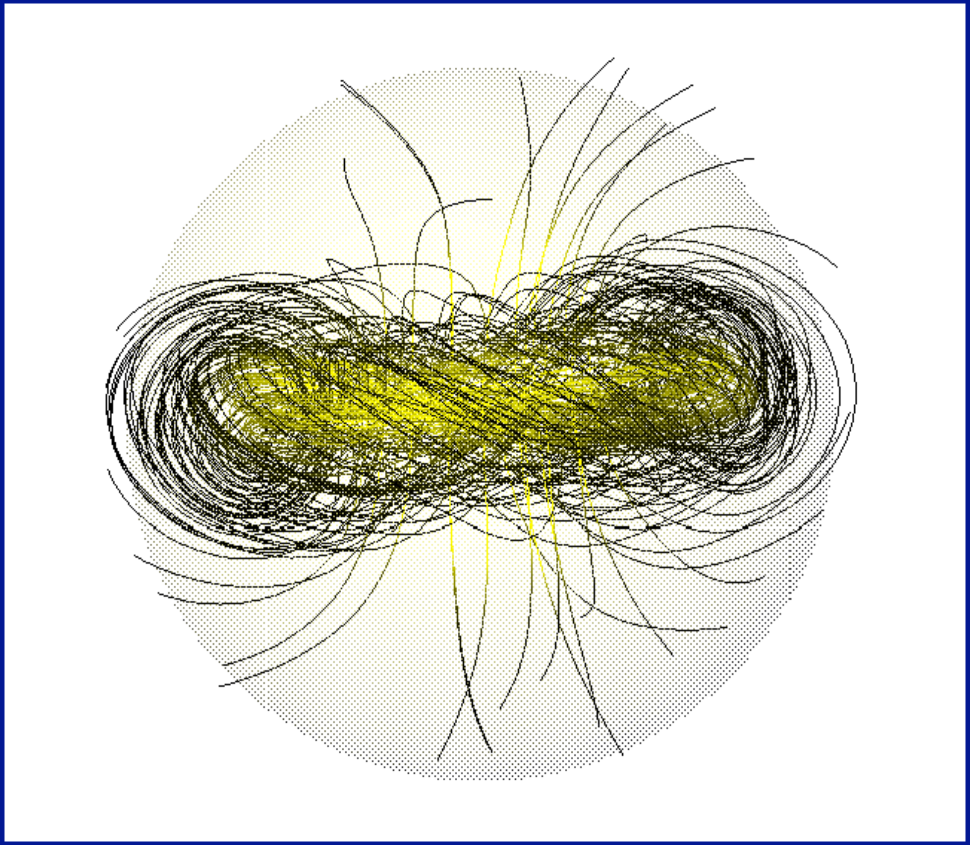
- Concept
  - Make numerical model of star with arbitrary magnetic field
  - Star has stable temperature profile
  - Follow evolution of field as it relaxes into an equilibrium
- First results
  - Simple axisymmetric equilibria are found (Braithwaite & Spruit 2004)

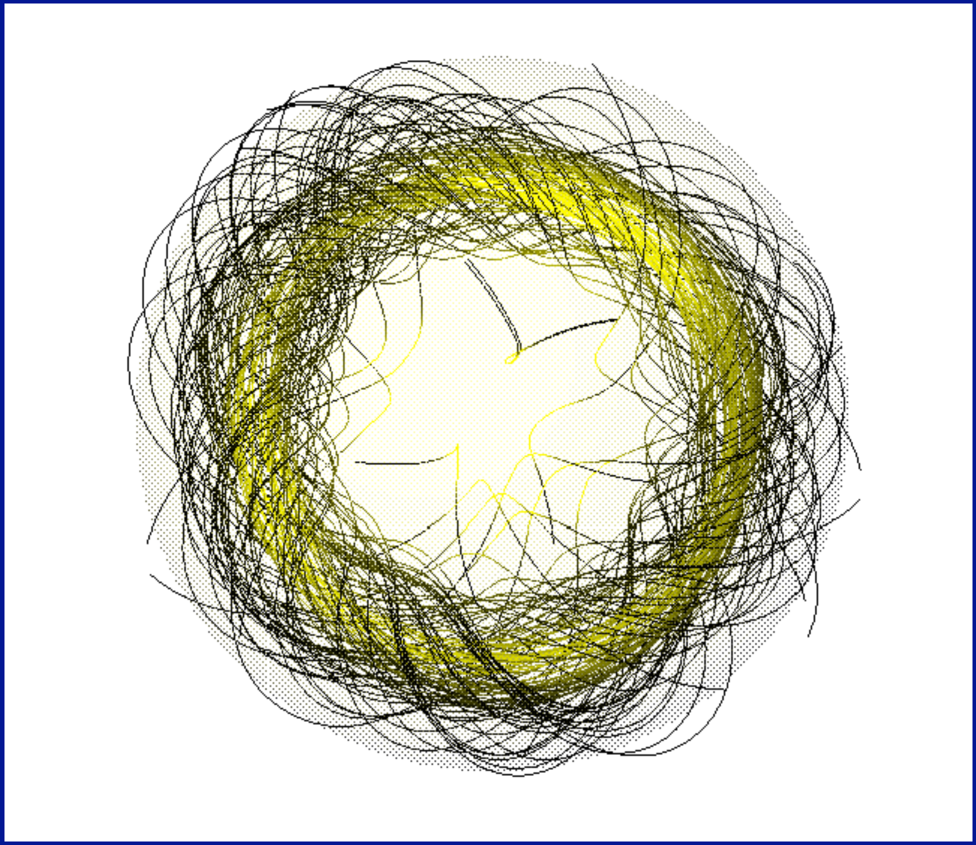


Formation of equilibrium



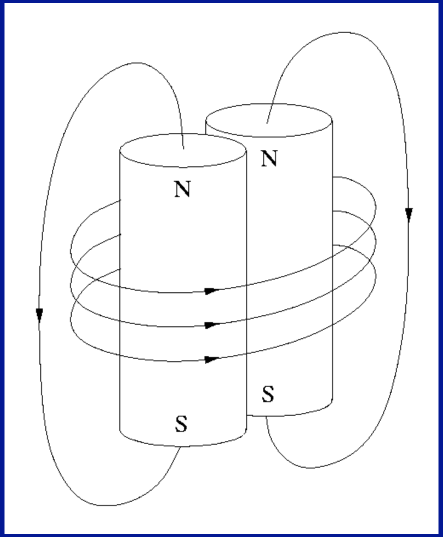
Most basic equilibrium





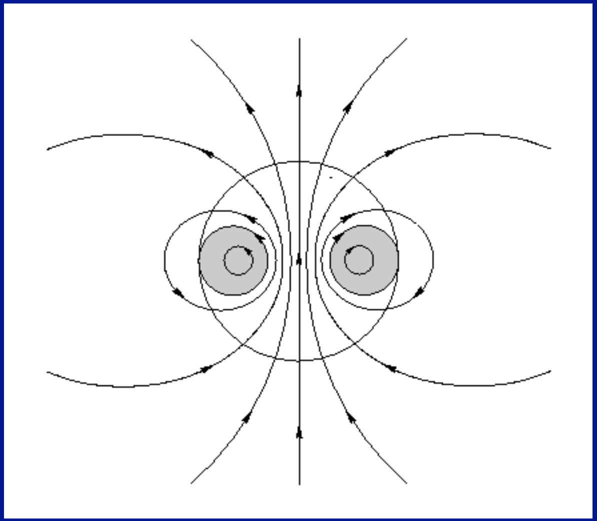
Braithwaite & Nordlund 2006

Stable equilibria

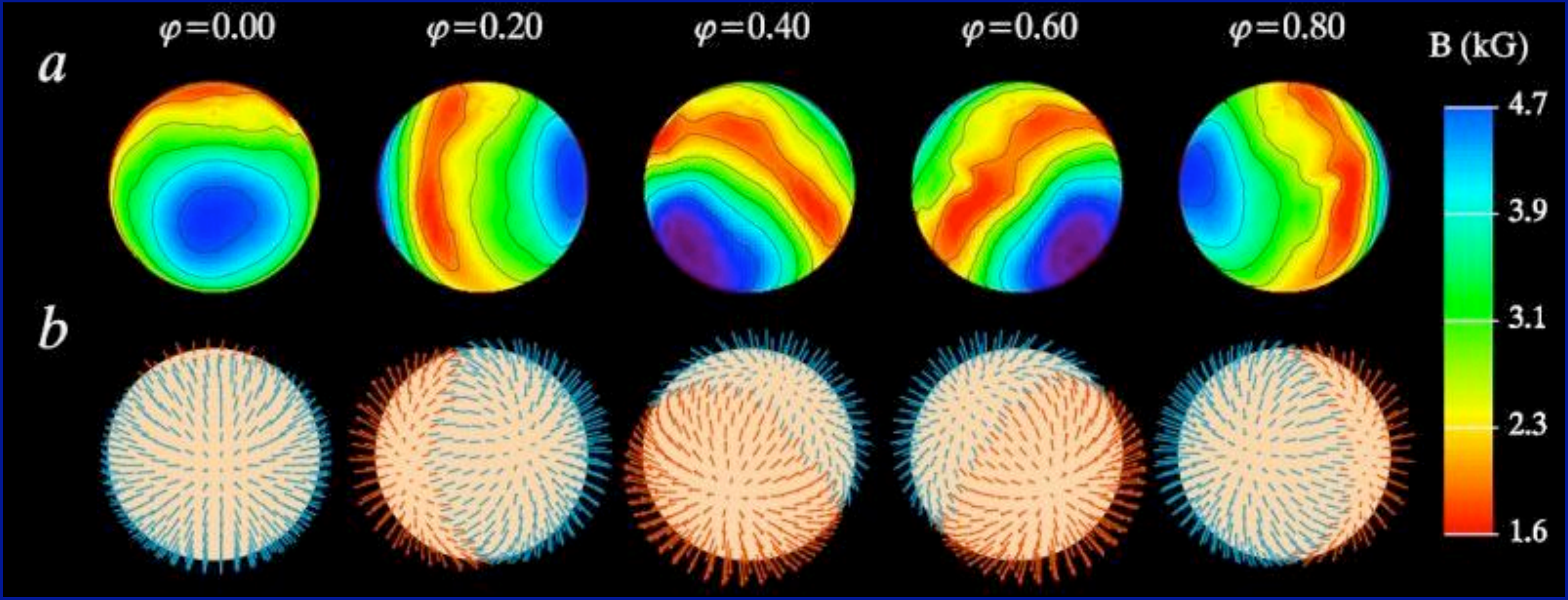


Simplest axisymmetric equilibrium consists of toroidal and poloidal components

(Prendergast 1956)



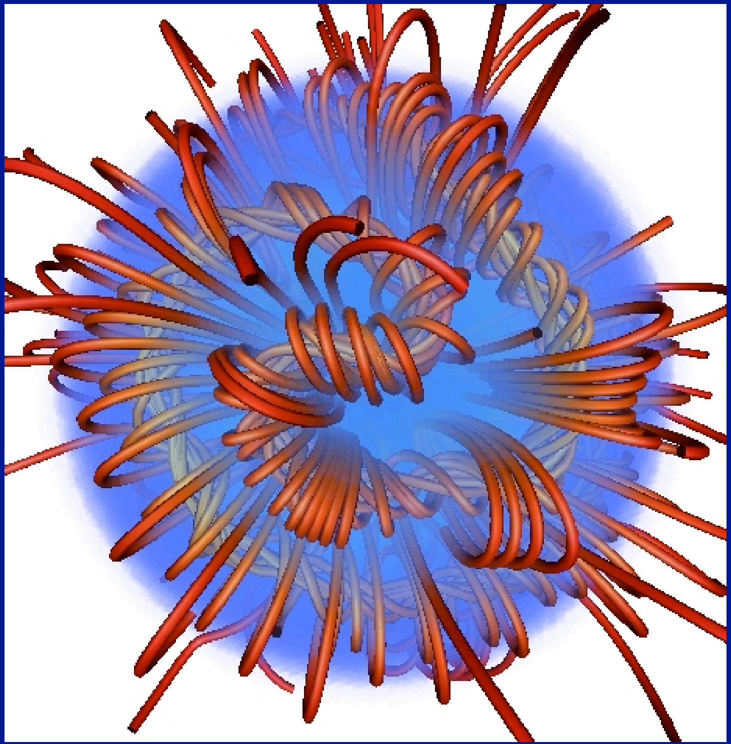
Comparison with observations

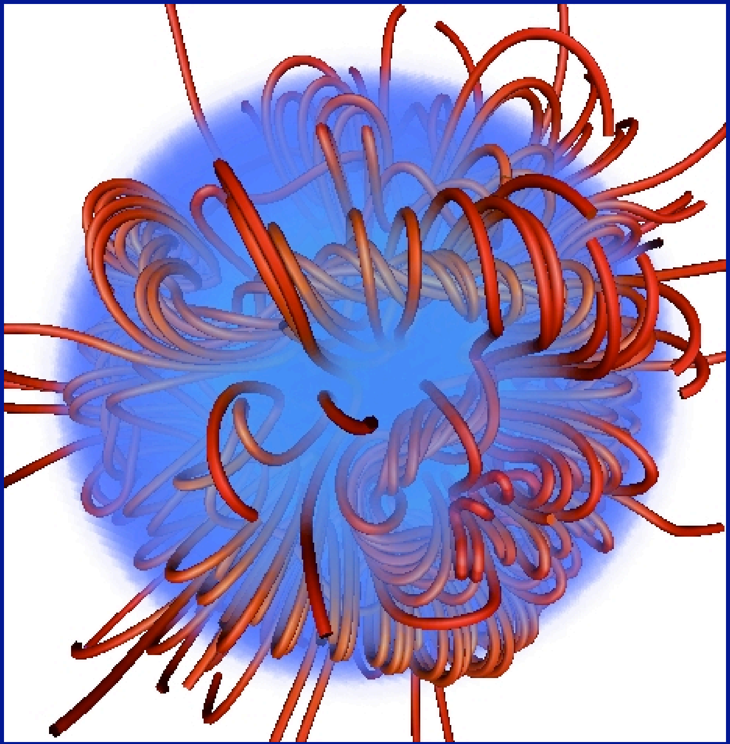


Field topology of α^2 CVn, an A-type main-sequence star
(Kochukhov et al. 2002, using Zeeman-Doppler imaging)

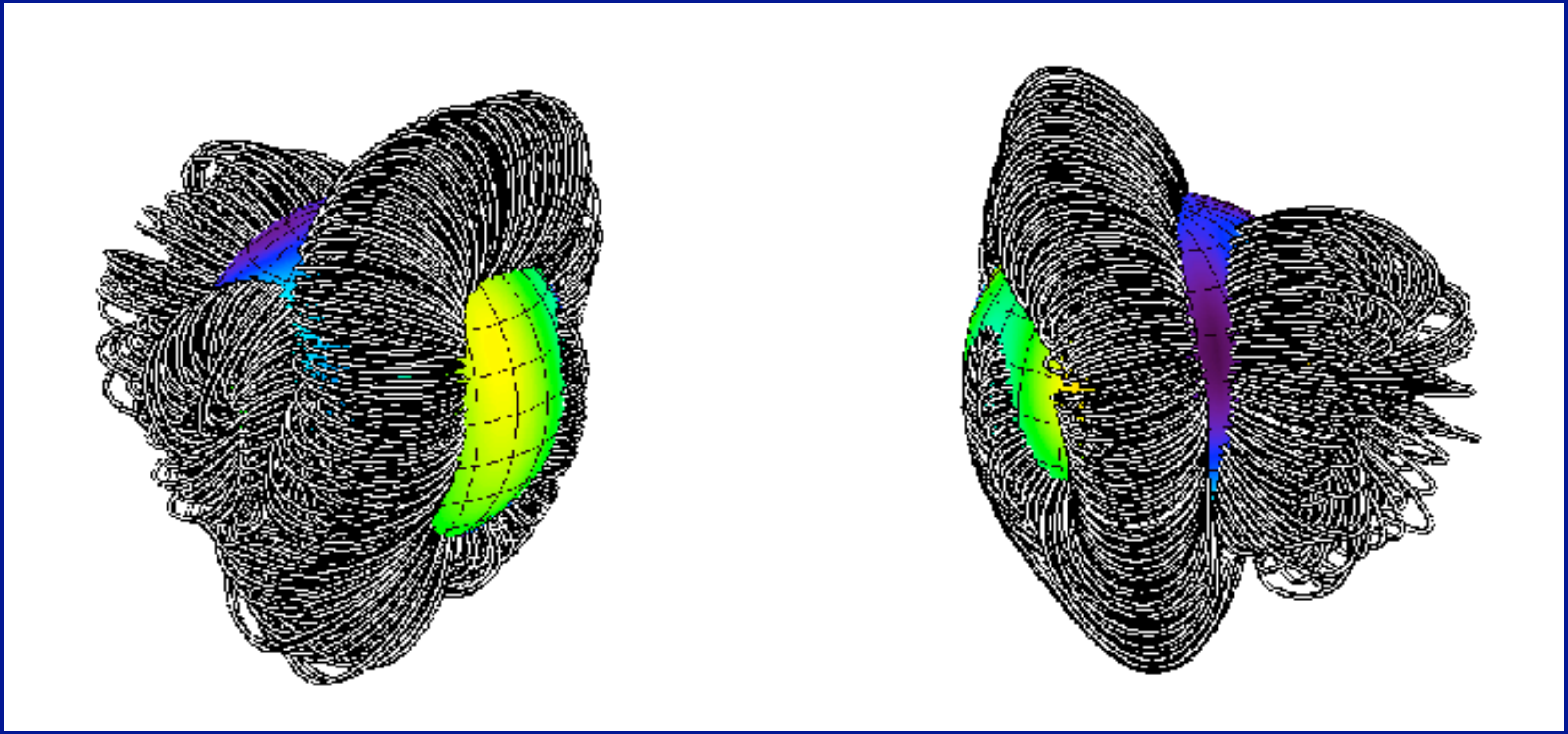
Other available equilibria...

Formed when initial field energy is less concentrated towards centre of star





Braithwaite 2008



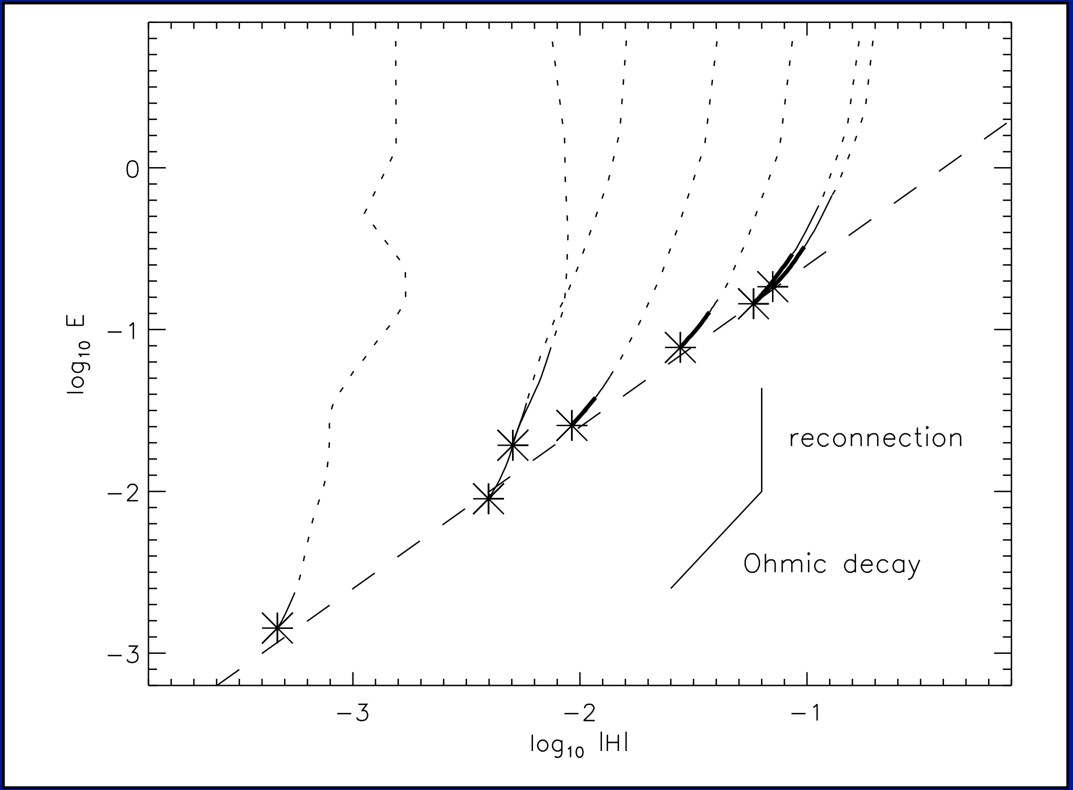
Field topology of τ Sco, a B0 main-sequence star (M_V=2.8)

(Donati et al. 2006, using Zeeman-Doppler imaging)

Energy and helicity in simulations

Braithwaite 2010

- Helicity falls only by ~20% during relaxation (improves with increasing resolution)
- Energy of equilibrium depends on helicity
- Same relation seen in stars and intergalactic bubbles



Log energy against log helicity
Solid lines represent equilibrium field
Straight dashed line shows H = λ R E where λ=0.3

Summary so far

- Poloidal flux during formation is not relevant parameter for strength of fields seen in upper-main-sequence
- Helicity is relevant parameter: $H \sim \Phi_{\rm pol}\Phi_{\rm tor}$
- Net toroidal flux and therefore helicity might be very small...? In most simulations, helicity is zero!

Change in helicity

- Pre-equilibrium helicity in protostar could be affected by
  - differential rotation --> Tayler-Spruit dynamo
  - convection
- Any change in helicity
  - happens on diffusive timescale
  - increase in (absolute value of) helicity requires some symmetry breaking
  - destruction of helicity does not, so is more likely

Importance of buoyancy

- Reconnection proceeds at Alfvén speed or slightly less
- In hydrostatically settled, isentropic star, magnetic field provides buoyancy because it gives pressure ($B^2/24\pi$) without mass
- In star with disordered field, regions of higher field strength are driven upwards
- This motion also happens at Alfvén speed:

$$\frac{\Delta\rho}{\rho} \sim \frac{\Delta P}{P} \sim \frac{B^2}{24\pi P} \sim \frac{v_\mathrm{A}^2}{c_\mathrm{s}^2} \qquad \text{and} \qquad F_\mathrm{buoy} \sim l^3 g \Delta\rho \sim \rho l^2 v_\mathrm{buoy}^2 \sim F_\mathrm{drag}$$

$$\text{and} \qquad H_p \approx \frac{c_\mathrm{s}^2}{g} \qquad \Rightarrow \qquad \frac{v_\mathrm{buoy}}{v_\mathrm{A}} \approx \left(\frac{l}{H_p}\right)^{1/2}$$

- End result: magnetic energy is lost into atmosphere, where helicity is *not* conserved

Simulations of relaxation in isentropic star

- Same simulation run as before, but star is polytrope of index n=3/2 instead of n=3
- Energy and helicity is lost into atmosphere

Braithwaite et al. in prep.

Summary

- Poloidal flux during formation is not relevant parameter for strength of fields seen in upper-main-sequence
- Helicity is relevant parameter: $H \sim \Phi_{\rm pol}\Phi_{\rm tor}$
- Net toroidal flux and therefore helicity might be very small...? In most simulations, helicity is zero!
- Helicity *might* change due to processes inside star, and only on long, diffusive timescale
- Helicity can be destroyed efficiently while star remains isentropic, i.e. convectively unstable
- Therefore lower-main-sequence stars naturally lose all their original helicity; magnetic properties are independent of initial conditions